Answers:

1. B
2. C
3. D
4. A
5. B
6. B
7. D
8. C
9. B
10. C
11. A
12. B
13. A
14. B
15. C
16. A
17. D
18. B
19. A
20. E (decrease of 10.9%)
21. B
22. C
23. E (3 or 3:1)
24. A
25. D
26. D
27. B
28. B
29. B
30. E $\left(\frac{81\sqrt{6}}{4}\right)$

1. Let the points of the triangle be A, T, and S. Alvin can either travel AT + TS or AS + ST. Using the 3-D distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$, we find AT = $\sqrt{69}$, AS = $\sqrt{66}$, and TS = $\sqrt{41}$. The minimum distance then is $\boldsymbol{\sqrt{66} + \sqrt{41}}$**, B.**

2. For any prism, the number of edges around the top face is equal to the number of edges around the bottom face, and is also equal to the number of edges stretching between the two bases. So there are 3 equal sets of edges, each containing 8 edges, meaning the bases are octagons. The solid then is constructed of two octagonal bases and the 8 faces connecting the two bases. 2+ 8 = **10, C.**

3. Converting each measurement to yards, the dimensions of the rectangular prism are, $\frac{4}{36}$ or $\frac{1}{9}$ yd., 1 yd., and $\frac{100}{3}$ yd. The volume is then $\frac{1}{9} \cdot 1 \cdot \frac{100}{3} = \frac{100}{27}$ which is more than $3\frac{1}{2}$ and less than 4. So order **4 yards, D.**

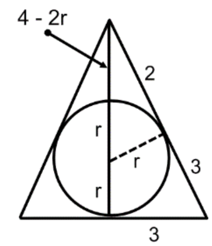
4. To approach this problem, I calculated the area of the front face, then I can just multiply that by the 12 feet that represent the uniform width. To Find the area, I broke it down into the cross-section of the deep end (a 10 by 8 rectangle), the shallow end (a 10 by 4 rectangle), and the ramped portion (a trapezoid with bases 4 and 8, and height 4). These have areas 80, 40, and 24, so the area of the front face is 144 square feet. When multiplied by the 12 ft. width, which is the height of the prism, the resulting volume is 1,728 cubic feet. If each cubic foot of water holds 7.5 gallons, just multiply 1,728 by 7.5 to get an answer of **12,960 gallons, A.**

5. We know from the radius that $x^2 + y^2 + z^2 = 13^2$. If I allow x to be 12, then I get $y^2 + z^2 = 25$, which has 12 solutions, namely: $(12, \pm5, 0)(12, 0, \pm5)(12, \pm3, \pm4), and\ (12, \pm4, \pm3)$. These 12 solutions have corresponding solutions when $y = 12$ or when $z = 12$. This gives a total of **36 satisfactory points, B.** (If we had also allowed a coordinate to be -12, then there would be 72 total points.)

6. There are 5 platonic solids – 3 made of only equilateral triangles, 1 made of only squares, and 1 made of only pentagons. It is simple enough to realize that 6 equilateral triangles can be arranged around a single point, but this would form a plane figure (a hexagon to be exact), so we must remove at least 1 of the triangles to begin to "fold" it into a 3-D figure. So, we could have 5 triangles around a point, and connect the edges on either side of the gap, we could have 4 triangles around a point, or we could have 3 triangles around a point. We cannot do less than 3 because we couldn't end up with a solid figure with FLAT faces. Then when forming a solid of squares, 4 squares would make up 360° around a point, so we remove one to have 3 squares (which begins to form a cube). Then 3 pentagons fill in 324° around a point, so we can fill the gap by folding. So there are **3 options** for the value of *k* (3, 4, or 5), **B.**

7. A sphere has radius 12. What is the ratio of its volume to its surface area? You can calculate the two values if you would like, but the ratio of volume to surface area for a sphere in general will be: $\frac{V}{SA} = \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{\frac{4}{3}r}{4} = \frac{r}{3}$ so for this example the ratio would be $\frac{12}{3}$ or **4:1, D.**

8. Consider the cross-section drawn. You will likely recognize that the height of the triangle is 4 since it is a 3-4-5 triangle. Also, the second 3 length is provided since tangent segments from the same point to the same circle are congruent. From there, I use similar triangles to get $\frac{r}{2} = \frac{3}{4}$. This leads the value of $r = \frac{3}{2}$. We can then find the length from the vertex to the circle to be **1, C.**



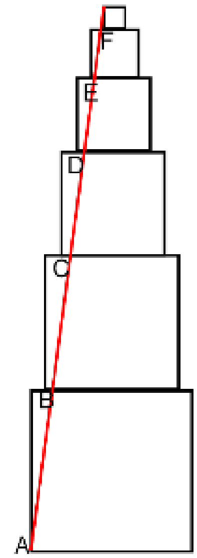
9. We begin the problem with some knowledge of two out of the three dimensions, which we will call $l, w, and\ h$. We will say we know $lw = 88$ since the area of a face is 88. We can also solve for h since we know the volume: $V = lwh = 88h = 1320$ so $h = 15$. We are looking for the diagonal of the prism and $d = \sqrt{l^2 + w^2 + h^2}$. The last piece of helpful information is that the perimeter of the given base is 48, so $2(l + w) = 48 \rightarrow l + w = 24 \rightarrow l^2 + 2lw + w^2 = 576$ and since $lw = 88$, then $l^2 + 2(88) + w^2 = 576 \rightarrow l^2 + w^2 = 400$. Now $d = \sqrt{(l^2 + w^2) + h^2} = \sqrt{400 + 15^2} = \sqrt{625} = \mathbf{25}$**, B.**

10. Two faces end up down in this problem, so we are trying to get a sum for all faces EXCEPT one on each die. Since each die sums to 21, or 42 for the pair, we can rephrase the question as "what is the probability that the two hidden faces have a sum less than 4." This only happens with a $(1,2), (2,1), or (1,1)$. There are 36 possible outcomes, so the answer is $\frac{3}{36} = \mathbf{\frac{1}{12}}$**, C.**

11. Find the volume of the cone. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{3}{2}\right)^2(4) = 3\pi$. So what will the required height of the cylindrical portion be in order to have a volume of 3π? $V = \pi r^2 h = \pi(3)^2 h = 3\pi$, and this gives $h = \mathbf{\frac{1}{3}}$**, A.**

12. Look at the tower from the top, and you will notice it is just a 6 by 6 square visible, and the same is true for the bottom. As far as lateral area goes, there are 4 lateral faces of each cube. This area is $4(1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2) = 4(91)$. The total surface are is $2(36) + 4(91) = \mathbf{436}$**, B.**

13. I just approach this from a two dimensional perspective, which keeps the radicals out of the work. I consider the slope of the red line, which I can calculate, since it rises 21 inches, and runs 2.5 inches. This gives $m_{red} = \frac{21}{2.5} = \frac{42}{5} = 8.4$. Now, I want to calculate the slope between points A and B, then between A and C, and so forth seeing when the slope of the red line is steeper than the slope of the line between A and some crucial point, meaning that the red line would pop outside the tower when it reaches the height of that point. Note: $m_{AB} = \frac{6}{0.5} = \frac{12}{1} = 12$; $m_{AC} = \frac{6+5}{0.5+0.5} = \frac{11}{1} = 11$; $m_{AD} = \frac{6+5+4}{0.5+0.5+0.5} = \frac{15}{1.5} = 10$; $m_{AE} = \frac{6+5+4+3}{4(0.5)} = \frac{18}{2} = 9$; $m_{AF} = \frac{6+5+4+3+2}{5(0.5)} = \frac{20}{2.5} = 8$ which is less than the slope of the red line. So the red line "pops out" of the tower with just one more unit to climb vertically. This means that 1 out of the 21 vertical units are exposed, and this will be true in all dimensions. So the answer is $\mathbf{\frac{1}{21}}$**, A.**



14. A quick sketch will reveal that the midsegment lies on the y-axis, and it has length 2. When the triangle is revolved around the y-axis, it will create a cylinder with two congruent cones missing (one from the top, and one from the bottom). A little work on equilateral triangles will say the height of the triangle is $2\sqrt{3}$, which means the radii of the cylinder and cones are all $\sqrt{3}$. The height of the cylinder is 4, the height of each cone is 1. So the volume of the generated solid is $V_{total} = V_{cylinder} - 2V_{cone} = \pi r^2 h - 2\left[\frac{1}{3}\pi r^2 h\right] = \pi\left(\sqrt{3}\right)^2(4) - 2\left[\frac{1}{3}\pi\left(\sqrt{3}\right)^2(1)\right] = 12\pi - 2(\pi) = \mathbf{10\pi}$**, B.**

15. The sector areas are in a ratio of 1:2:3, which will tell us they are 60°, 120°, 180° sectors. So the largest is a semicircle. The arc length of this circle is $\frac{180}{360} \cdot 2\pi(6) = 6\pi$. When this forms the cone, the circumference of the base of the cone will be 6π, which means the radius of the cone will be 3. The radius of the original circle becomes the slant height of the cone, which is 6. Then using the Pythagorean Theorem to find the height, we get $h = 3\sqrt{3}$. The volume is $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)^2\left(3\sqrt{3}\right) = \mathbf{9\pi\sqrt{3}}$**, C.**

16. This is done by folding along the 3 midsegments of the triangle, which mades each face of the tetrahedron an equilateral triangle with side length 6. The base area will then be $9\sqrt{3}$, and the height can be found as $2\sqrt{6}$ by using the circumradius of the base is $2\sqrt{3}$ and the lateral edge of the tetrahedron is 6. The $V = \frac{1}{3}Bh = \frac{1}{3}\left(9\sqrt{3}\right)\left(2\sqrt{6}\right) = \mathbf{18\sqrt{2}}$**, A.**

17. The total area is 6 square faces, while the lateral area is 4 square faces. Thus the lateral area is $\frac{4}{6}$ or $\frac{2}{3}$ of the 324 square inch surface area… **216, D.**

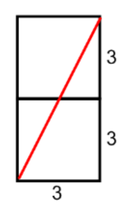
18. A square pyramid has a base edge of length 10 and a lateral edge of length 13. What is the volume of the pyramid? We will need the height of the pyramid. A lateral edge, a slant height, and half of a base edge form a 5-12-13 right triangle, so the slant height is 12. Then we find the height of the pyramid by using the slant height as the hypotenuse and half of the base edge as one leg to get $h = \sqrt{119}$. $V = \frac{1}{3}Bh = \frac{1}{3}(10^2)(\sqrt{119}) = \mathbf{\frac{100\sqrt{119}}{3}}$**, B.**

19. A right circular cone has total area of 144π and a lateral area of 80π. What is the volume of the cone? Since total area for a cone is lateral area plus base area, we know the base area is 64π. This means the radius is 8. Since the lateral area is πrl, and we know the radius, we can find that $l = 10$. This enables us to find the height of the cone to be 6. Then . $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(8)^2(6) = \mathbf{128\pi}$**, A.**

20. A right circular cone is altered so that its height is increased by 10 percent and its radius is decreased by 10 percent. What impact has this made on the volume of the cone (to the nearest tenth of a percent)? $V = \frac{1}{3}\pi r^2 h$, but with the changes, it looks like $\frac{1}{3}\pi(0.9r)^2(1.1h) = (0.9)^2(1.1)\left[\frac{1}{3}\pi r^2 h\right] = 0.891\left[\frac{1}{3}\pi r^2 h\right]$ which shows a **decrease of 10.9%, E.**

21. The ratio of volumes is $a^3 : b^3 = 250 : 128 = 125 : 64$ which gives $a : b = 5 : 4$, the linear ratio. Then the area ratio for the two solids is $a^2 : b^2 = 25 : 16$. Since we know the SA of one of the solids, we solve $\frac{25}{16} = \frac{150}{x}$, so $\boldsymbol{x} = \mathbf{96}$**, B.**

22. A quick sketch will show that the ant only has to crawl on two faces, but he will have to go 3 units in one direction while going 6 units another direction, so his straight path will be $\sqrt{3^2 + 6^2} = \sqrt{45} = \mathbf{3\sqrt{5}}$**, C.**



23. What is the ratio of the volume of the cylinder to the volume of the double cone? Just write the ratio, then tons of simplification happens. $\frac{V_{cyl}}{v_{2cones}} = \frac{\pi r^2 h_1}{2\left[\frac{1}{3}\pi r^2 h_2\right]}$ since the radii are the same, we have $\frac{h_1}{2\left[\frac{1}{3}h_2\right]} = \frac{8}{2\left[\frac{1}{3}\cdot 4\right]} = \frac{1}{\frac{1}{3}} = \mathbf{3}$**, E.**

24. A solid white cube of side length 5 is dipped into blue paint and then cut into separate 1 inch cubes. What is the probability that a randomly drawn 1 inch cube will be painted blue on exactly 2 sides? When the cube is cut up, there are $5^3 = 125$ smaller cubes. The ones of these that are painted on 2 sides are the ones that are along the edges, but not on the corners, which gives 3 along each of 12 edges, or 36. The probability is then $\mathbf{\frac{36}{125}}$**, A.**

25. The volume of the original pyramid is $V = \frac{1}{3}Bh = \frac{1}{3}(12^2)(8) = 384$. Since we are cutting half way up the height, and using ratio's of similar solids, the small pyramid removed from the top is $\frac{1}{8}$ of the volume of the original, or 48. The frustum represents the remaining $\frac{7}{8}$, or **336, D.**

26. Larry must crawl 3 + 4 + 5 or 12 total units. The way these are split up among the 3 directions can vary, so we are looking for how many ways we can take 12 moves, and choose 3 in the x-direction, 4 in the y-direction, and 5 in the z-direction. This is $12C3 \cdot 9C4 \cdot 5C5 = \frac{12!}{3!9!} \cdot \frac{9!}{4!5!} \cdot \frac{5!}{5!0!} = \frac{12!}{3!4!5!} = \frac{12\cdot11\cdot10\cdot9\cdot8\cdot7\cdot6}{3\cdot2\cdot1\cdot4\cdot3\cdot2\cdot1} = 11 \cdot 5 \cdot 9 \cdot 8 \cdot 7 = \mathbf{27,720}$**, D.**

27. I considered this with a type of "vector" thinking. He will travel 5 units in one direction (say right), 4 units in a second direction(say up), and 3 units in a third direction (say back). I broke this down into 5 segments, thinking about how he would travel $\langle 1, \frac{4}{5}, \frac{3}{5} \rangle$ each increment, and each of these motions would take him 1 unit right, 4/5 up, and 3/5 back. Notice, this first segment takes him to the point $\left(1, \frac{4}{5}, \frac{3}{5}\right)$ is contained in just one block. However, the successive moves are slightly more involved. A second motion would take him to the point $\left(2, \frac{8}{5}, \frac{6}{5}\right)$, so notice as he travels through this second layer, he also passes a coordinate of "1" in the up direction and the back direction, taking him through not just one block this time, but two additional blocks, so 3 in this layer. The next move goes to $\left(3, \frac{12}{5}, \frac{9}{5}\right)$, which makes two of the coordinates reach their next integer value, so we pass through 2 blocks. Using symmetry, or the same methods employed here, we will see that the last two layers are just the reverse of the first two layer (think about if you had started from the other end). So, in the five layers, Larry passes through 1 + 3 + 2 + 3 +1 or **10 cubes, B.**

28. What is the smallest number of additional cubes it would take to construct a protective layer of 1 inch cubes around the original prism so that the resulting solid is also a prism? Basically, just turn the 3 by 4 by 5 into a 5 by 6 by 7. Then $7 \cdot 6 \cdot 5 - 5 \cdot 4 \cdot 3 = \mathbf{150}$**, B.**

29. A water tank in the shape of a cylinder with radius 4 feet and height 9 feet is laying on its side. If the water in the tank is 2 feet deep, then find the volume of water in the tank, in cubic feet. Note the cross-section diagram given here. Since the radius is 4 and the depth is two, this leads us to find that the right triangles have a leg of 2, and ultimately to discover that they are 30°-60°-90° triangles, and that the sector of the circle which contains the cross-section of the water is a 120° sector. Furthermore, you can find the other leg of the triangle to be $2\sqrt{3}$ and the length of the blue segment is $4\sqrt{3}$. So the area of the water in the diagram is $A_{sector} - A_{triangle} = \frac{120}{360}\pi(4)^2 - \frac{1}{2}\left(4\sqrt{3}\right)(2) = \frac{16\pi}{3} - 4\sqrt{3}$. When this is multiplied by the height of the cylinder we have $9\left[\frac{16\pi}{3} - 4\sqrt{3}\right] = \mathbf{48\pi - 36\sqrt{3}}$**, B.**

30. The volume and surface area of a hemisphere are numerically equivalent. What is the volume of the largest cube which can be inscribed in this hemisphere? Note that $V_{sphere} = \frac{4}{3}\pi r^3$ so $V_{hemisphere} = \frac{2}{3}\pi r^3$. Also $SA_{sphere} = 4\pi r^2$, and when it is cut in half, you have half of the outside of the sphere, plus the great circle that is now visible, $SA_{hemisphere} = 2\pi r^2 + \pi r^2 = 3\pi r^2$. So we want to find the value of r where $\frac{2}{3}\pi r^3 = 3\pi r^2$ which gives $r = \frac{9}{2}$. Now, inscribe the cube. The center of the bottom face of the cube will be at the center of the great circle. Also note that the radius of the sphere will extend from the center of this face to any of the vertices on the opposite face of the cube. I will let the edge of the cube be $2x$ for simplicity. This means to get from the center of the bottom face to a vertex on the top face, I might travel x units right, x units back, and $2x$ units up. Then finding the length of this radius in terms of x, $r = \sqrt{x^2 + x^2 + (2x)^2} \rightarrow \frac{9}{2} = \sqrt{6x^2} \rightarrow \frac{81}{4} = 6x^2 \rightarrow x = \frac{3\sqrt{3}}{2\sqrt{2}}$ which means the side of the cube is $\frac{3\sqrt{3}}{\sqrt{2}}$ and the volume would be $\left(\frac{3\sqrt{3}}{\sqrt{2}}\right)^3 = \frac{81\sqrt{3}}{2\sqrt{2}} = \boldsymbol{\frac{81\sqrt{6}}{4}}$**, E.**